ATMOSPHERIC RE-ENTRY INTO THE MARTIAN ATMOSPHERE WITH VARIABLE LIFT CONTROL

Hiroshi Ohtakay

California Institute of Technology+

Pasadena, California

Abstract

A gliding type re-entry of a space capsule into the Martian atmosphere for soft-landing with the minimum heat generation is investigated. A nonlinear filter which can be installed on-board is developed to estimate the unknown atmospheric parameter values of Mars in real-time. A stochastic control problem with state variables estimation is investigated for the re-entry capsule which is subject to random movements of the Martian atmosphere due to wind and wind gusts.

Introduction

The Martian atmosphere[1] is believed to be considerably thinner than that of the earth by a factor which ranges from 50 to 300. The precise Martian atmospheric density value is not known. Therefore, a gliding type re-entry capsule should have a capability of adapting its flight path to the unknown atmospheric characteristics.

While the capsule is at high altitudes (higher than $100. \times 10^3$ ft), the random movement of the atmosphere is negligible, compared with the speed of the capsule (the re-entry speed is assumed to be 16.0×10^3 ft/sec). On the other hand, the random movement of the Martian atmosphere has a significant effect on the flight of the capsule at altitudes below $100. \times 10^3$ ft. Maximum wind and wind gusts speeds up to 400 ft/sec may occur (see reference [1]). The speed of the capsule at the time of parachute ejection (at an altitude of 15.0×10^3 ft) is desired[2] to be a little less than 1000. ft/sec. A guidance and control system which can cope with the random disturbance torques would make a major contribution to the successful softlanding of the capsule in a predetermined area of the Martian surface. Such a stochastic guidance and control problem is investigated separately from the atmospheric parameter estimation problem by solving the latter before the capsule comes down to low altitudes.

Deterministic Optimal Control Problem

The planar motion of the space capsule is characterized by the following set of equations (see reference [2]):

$$\frac{dX_1}{dt} = k_1 \sin X_2 - \frac{\rho(X_3)}{2k_2} X_1^2 \qquad (1)$$

$$\frac{dX_2}{dt} = \left(\frac{k_1}{X_1} - \frac{X_1}{k_3}\right) \cos X_2 - \frac{L}{D}\frac{\rho(X_3)}{2k_2} X_1 \qquad (2)$$

$$\frac{dX_3}{dt} = -X_1 \sin X_2 \qquad (3)$$

where:

$X_1 \triangleq$ speed of the capsule

$X_2 \triangleq$ flight path angle

$X_3 \triangleq$ altitude

$k_1 \triangleq$ gravitational constant of Mars, 12.3 ft/sec^2

$k_2 \triangleq$ ballistic constant, 0.4 slug/ft^2

$k_3 \triangleq$ radius of Mars, 11.2×10^6 ft

$\rho(X_3) \triangleq k_4 \operatorname{Exp}[-k_5 X_3]$, atmospheric density at altitude X_3

$k_4, k_5 \triangleq$ atmospheric parameters of Mars

$L/D \triangleq$ gasdynamic lift to drag ratio; control force

The performance index of the re-entry trajectory which minimizes the heat generation, denoted by Q_T, due to convection at the nose of the capsule is given by (see reference [2]):

$$Q_T \triangleq \int_{t_e}^{t_f} 20.4 \times 10^{-9}\, k_6^{-\frac{1}{2}}\, X_1^3\, \rho^{\frac{1}{2}}(X_3)dt \text{ BTU/ft}^2 \qquad (4)$$

where:

$t_e \triangleq$ capsule's impact time on the Martian atmosphere

$t_f \triangleq$ time of a parachute ejection

$k_6 \triangleq$ nose radius of the capsule, 1.0 ft

It is known[3] from wind tunnel experiments that a large value of L/D is very difficult to achieve. Thus, the control force is bound in magnitude:

$$|L/D| \le u_o \qquad (5)$$

where $u_o = 0.6$ in this investigation.

In order to prevent the capsule from impacting the Martian surface, an inequality constraint

$$X_3(t) \ge k_7$$

is also assumed for $t_e \le t \le t_f$, where:

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$k_7 \triangleq$ altitude of a parachute ejection, 15.0×10^3 ft

The control law derived for this problem is of the bang-bang type, with two switching boundaries (see reference [2]) denoted by

$$\psi_1(\underline{X},\underline{k}) = 0 \text{ and } \psi_2(\underline{X},\underline{k}) = 0,$$

where:

$$\psi_1 \cong 1 + \frac{u_o}{2k_2k_5}\{\rho(X_3) - \rho(k_8)\} - \cos X_2 \qquad (7)$$

$$\psi_2 \cong \cos\{X_2 + k_2k_5 Z \ln(\frac{X_{1f}}{X_1})^2\} + \frac{Z}{2}\{\rho(X_3) - \rho(k_8)\} - \cos X_2 \qquad (8)$$

$$Z \triangleq -\frac{u_o}{2}\frac{1}{k_2k_5} - \frac{\cos X_2}{k_2k_5\,\rho(X_3)}\left(\frac{k_1k_3}{X_1^2} - 1\right) \qquad (9)$$

$X_{1f} \triangleq$ speed of the capsule when a parachute is ejected

"The second order theory of re-entry mechanics"[4] is utilized in obtaining the above approximate analytic solutions. Typical re-entry trajectories are depicted in Fig. 1.

Estimation of System Parameters

A shallow re-entry angle corridor makes it difficult to utilize a radar observation for finding the gliding path of the capsule [5] (which has a slant radar, mounted along the roll axis). Thus, the inertial observation is the only available source of flight path information. Such observation is corrupted by noise due to orbit determination error, gyro measurement errors at bus-capsule separation and dynamical errors in accelerometers, during the flight of the capsule in high altitudes. For the flight of the capsule in low altitudes, Martian wind and wind gusts are added to the above noise sources. Fig. 2 illustrates the results of a computer simulation of typical noise processes in the inertial observation. It is seen that they are biased and highly correlated.

A sequential filter[6] which minimizes the quantity (10)

$$||\underline{X}(t_e) - \underline{\mu}_e||^2_{\underline{P}^{-1}_{(0)}} + \int_{t_e}^{t}\{||\epsilon_1(\tau)||^2_{Q_{(\tau)}} + ||\epsilon_2(\tau)||^2_{\underline{P}_{(\tau)}}\}d\tau$$

for each instant of time t is developed, where:

$\underline{\mu}_e$ = a priori knowledge of the vector $\underline{X}(t_e)$

$Q(\cdot), P(\cdot)$ = weighting matrices which are at least positive semi-definite and definite, respectively

$\epsilon_1(\cdot), \epsilon_2(\cdot)$ = residual errors in observation and in flight dynamics, respectively

The prime reason for employing this filter is that it does not require any particular statistical assumptions of the noise processes involved. A typical filter performance is shown in Fig. 3. The closed form solutions, i.e., eqs. (7) and (8), of the deterministic optimal control problem and the successful parameter identification shown in Fig. 3 complete the adaptive control system for a space capsule re-entering into the partially known atmosphere of Mars.

Stochastic Optimal Control Problem

In order to investigate a guidance problem of the space capsule under the disturbance of random movements of the Martian atmosphere, a mathematical description of the wind and wind gusts characteristics is necessary. This is done (see reference [2]) by utilizing Ormstein-Uhlenbeck process[7]:

$$d\zeta = -k_8\,\zeta\,dt + dw \qquad (11)$$

where

$k_8^{-1} \triangleq$ correlation time

$dw/dt \triangleq$ Gaussian white noise process

A set of thrusts (for example, cold gas reaction jets) along the roll and yaw axes of the capsule are assumed to be available for adjusting the flight path of the capsule. The guidance problem can be solved within the context of stochastic optimal control theory of a linear system[8] since a nominal trajectory, denoted by $\underline{X}^{(N)}(t)$, can be defined based upon the estimated values of the Martian atmospheric parameters. That is, the flight path of the capsule is characterized by:

$$d(\delta\underline{X}) = \{A(t)\,\delta\underline{X} + B(t)\upsilon\}\,dt + d\underline{w} \qquad (12)$$

where

$$\delta\underline{X}(t) \triangleq \underline{X}(t) - \underline{X}^{(N)}(t) \qquad (13)$$

$$A(t) \triangleq \left(\frac{\partial f}{\partial X}\right)_{X=X(N)}$$

$$B(t) \triangleq \begin{array}{c} \begin{matrix} 1 & 0 & 0 \\ 0 & X_1^{(N)} & 0 \end{matrix} \\ \hline 0_{33} \end{array}$$

The problem is formulated in such a way that the quantity

$$E[||C\underline{X}(t_f)||^2_R + \int_{t_n}^{t_f} ||\upsilon||^2_{S(\tau)}d\tau] \qquad (14)$$

is minimized, where:

$E[\cdot] \triangleq$ expectation of $[\cdot]$

$||\upsilon||^2 \triangleq$ energy associated with the thrust vector

$S, R(\cdot) \triangleq$ weighting matrices which are at least positive and semi-positive definite, respectively

The optimal guidance law, denoted by $\bar{\upsilon}(t)$, is found to be

$$\bar{\upsilon}(t) = -S^{-1}B^{*}F(t)E[\delta\underline{X}] \qquad (15)$$

where * means transpose of a matrix. The feedback gain matrix F(t) is given by the solution of the following Riccati eq. (see reference [2]):

$$- \frac{dF}{dt} = FA + A^*F - F\,(BS^{-1}B^*)\,F \qquad (16)$$

with the boundary condition

$$F(t_f) = R \qquad (17)$$

The feasibility of the resulting stochastic optimal guidance law with state variables estimation is verified by a Monte Carlo simulation. Table 1 shows energies associated with the thrusts for eight different sample functions of wind and wind gust model. A set of typical thrust programs are depicted in Fig. 5.

Conclusion

The feasibility of a gliding type re-entry of a space capsule into the Martian atmosphere for softlanding is demonstrated. The approximate closed loop solution of the deterministic optimal control problem is justified by an open-loop solution (see Fig. 1). A nonlinear filter which enables a sequential estimation of the state of the re-entry dynamics as well as the unknown parameters is developed. A simplification of such a filter is achieved without degrading the resulting filter performance (see Fig. 3). It is also shown that a set of relatively small thrust vectors (see Fig. 5) can guide the capsule within 3% deviation at the target, even though the capsule is under random disturbance torques due to wind and wind gusts.

References

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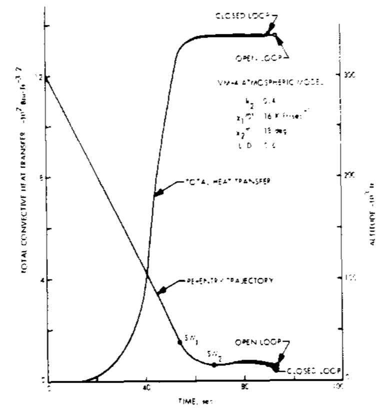

Fig. 1 Open and closed-loop trajectories and the corresponding performance indices.

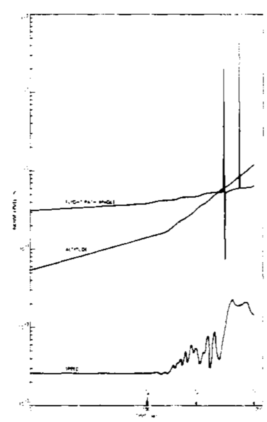

Fig. 2 Noise level in inertial observation.

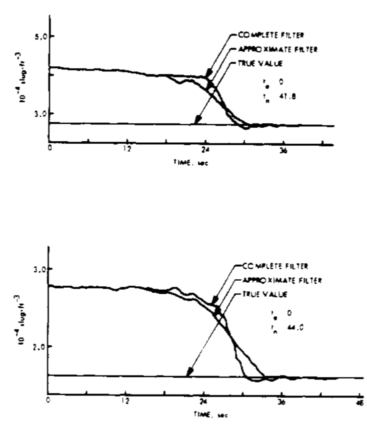

Fig. 3 A typical filter performance for VM-4 and VM-8 atmospheric models.

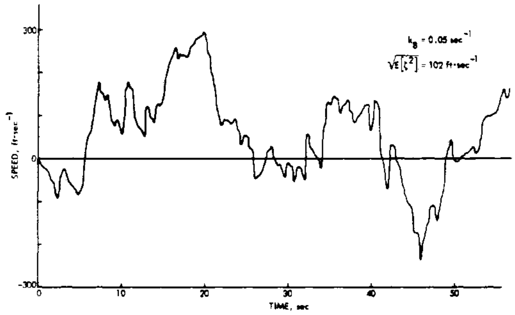

Fig. 4 A Martian wind and wind gusts model.

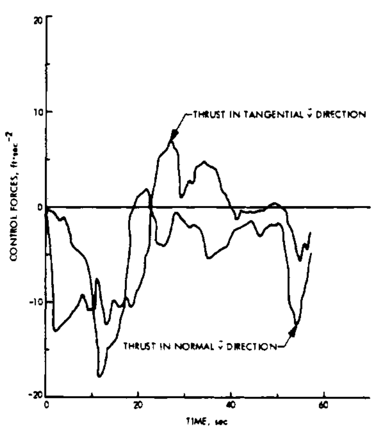

Fig. 5 A set of typical thrust programs.

Energy consumption of thrusts for different sample functions of white noise processes: The values represent the quantity $\int_{t_n}^{t_f} ||v||_S^2 dt$

Sample Number	Case I	Case II
1	.27064 x 10^4	.26908 x 10^4
2	.26784	.26800
3	.27298	.27300
4	.26787	.27288
5	.27084	.27084
6	.26827	.26828
7	.26996	.27719
8	.26817	.26812

Case I: 0% Deviation in Speed and Flight Path Angle at $t = t_n$.

Case II: 5% Deviation in Speed and Flight Path Angle at $t = t_n$.

Table 1 Monte Carlo simulation of thrust energies